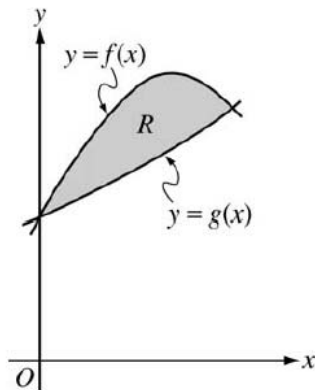


Volumes of Revolution and of Solids with Known Cross-Sections

1. (Calculator)



Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

- Find the area of R .
- Find the volume of the solid generated when R is revolved about the x -axis.
- The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

2. (Calculator)

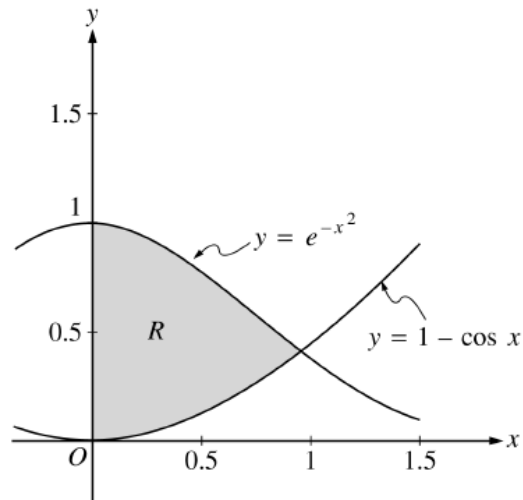
Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x -axis.

(a) Find the area of R .

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

(c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

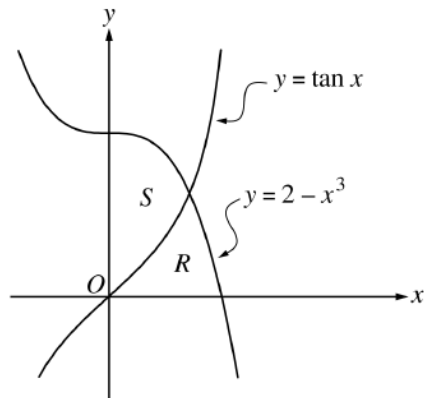
3. (Calculator)



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y -axis, as shown in the figure above.

- Find the area of the region R .
- Find the volume of the solid generated when the region R is revolved about the x -axis.
- The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.

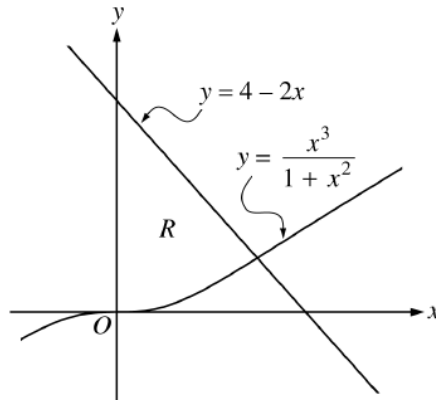
4. (Calculator)



Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x -axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y -axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

- (a) Find the area of R .
- (b) Find the area of S .
- (c) Find the volume of the solid generated when S is revolved about the x -axis.

5. (Calculator)



Let R be the region bounded by the y -axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

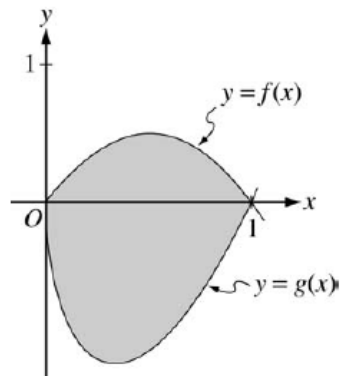
- Find the area of R .
- Find the volume of the solid generated when R is revolved about the x -axis.
- The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.

6. (Calculator)

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

- (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.
- (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.
- (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \leq x \leq 1$. Show the analysis that leads to your answers.

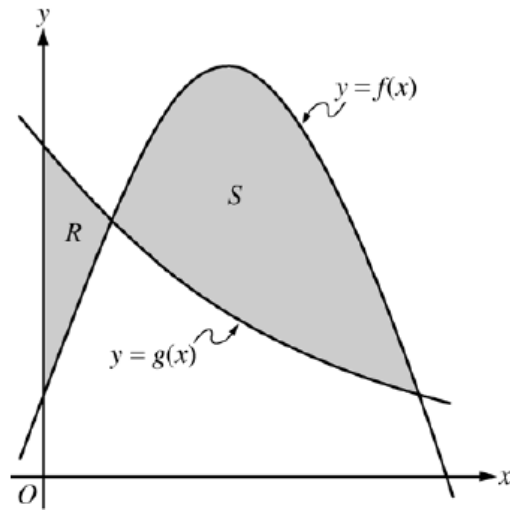
7. (Calculator)



Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \leq x \leq 1$. The graphs of f and g are shown in the figure above.

- Find the area of the shaded region enclosed by the graphs of f and g .
- Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.
- Let h be the function given by $h(x) = kx(1-x)$ for $0 \leq x \leq 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x -axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k .

8. (Calculator)



Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.

- Find the area of R .
- Find the area of S .
- Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.